

# Is $Z_b(10610)$ a Molecular State?

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## Abstract

Whether molecular states indeed exist in nature has been disputed for a long time. Several new resonances have been observed in the recent experiments and they seem to be of exotic structures and some of them have been proposed to be molecular states. The very recent observation of  $Z_b(10610)[(10608.4 \pm 2.0) \text{ MeV}]$  and  $Z_b(10650)[(10653.2 \pm 1.5) \text{ MeV}]$  encourages the interpretation of multi-quark states. In the Bethe-Salpeter (BS) approach, we study the possibility if two heavy mesons can form a molecular state by exchanging light mesons. Our results indicate that two heavy mesons can form an isospin singlet ( $I = 0$ ) bound state but cannot form an isospin triplet ( $I = 1$ ) when the contribution of  $\sigma$ - exchange is reasonably small, i.e. as the coupling of  $\sigma$  with mesons  $g_\sigma$  takes the value given in previous literatures. Thus we conclude that the newly observed  $Z_b(10610)$  should not be a molecular state, but a tetraquark state instead, at most, the fraction of the molecular state in the physical resonance  $Z_b(10610)$  is tiny.

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## I. INTRODUCTION

The naive non-relativistic quark model[1] tells us that a meson contains a quark and an anti-quark while a baryon is composed of three valence quarks. However, it is also noted that the exotic hybrid, glueball and multi-quark states are not excluded by the  $SU(3)$  quark model. Until now no any meson or baryon has been confirmed as an exotic state even though several resonances are considered to be exotic, namely have different component-structure from regular hadrons. There is not any principle to forbid their existence, and in fact the discussion about the exotic states has never stopped. For example the exotic structure of  $f_0(980)$  and  $a_0(980)$  is still under intense dispute [2–6]. Since the conventional  $q\bar{q}$  structure cannot well fit experimental data of certain modes[7, 8], some authors proposed that they may be molecular states made of two color-singlet mesons or tetraquarks composed of a color-anti-triplet diquark and a color-triplet anti-diquark [2–4], .

Recently, a series of charmonium-like resonances have successively been experimentally observed, such as  $X(3872)$ [9],  $X(3940)$ [10],  $Y(3940)$ [11],  $Z(4430)^\pm$ [12] and several bottomonium-like states are also discovered by the Belle and Babar collaborations, such as  $Z_b(10610)$  and  $Z_b(10650)$  [13]. It is noted that there is almost no room in the regular representations of  $O(3) \otimes SU_f(3) \otimes SU_s(2)$  to accommodate those newly observed resonances. Especially these particles  $Z(4430)$ ,  $Z_b(10610)$  and  $Z_b(10650)$  with non-zero charge cannot be understood in the conventional  $Q\bar{Q}$  structure if they are real resonances [14–18]. Concretely the mass of  $Z_b$  is between the mass of  $\Upsilon(4S)$  and  $\Upsilon(5S)$ , so it should have one  $b$  (quark) and one  $\bar{b}$  (anti-quark) but it by no means is a bottomonium because of its electric charge. Apparently if it is verified to be a resonance it should be a multi-quark state, namely a molecular state or tetraquark should be the preferred choice since they are the simplest extension beyond the regular  $Q\bar{Q}$  structure.

Guo et al. explored possible  $K\bar{K}$  bound states in the Bethe-Salpeter framework and found that the bound state could exist [19]. In the same framework the resonance  $B_{s0}^*(5725)$  was considered as a  $B\bar{K}$  molecular state[20]. Whether two heavy mesons can form a molecular state has not been thoroughly investigated in those works[19, 20]. Thus in this work we will try to study the question in the Bethe-Salpeter (BS) approach where the relativistic corrections are automatically included. Besides the bound states of two pseudoscalars we also explore the bound states which contain one or two vectors. For the aim we need to deduce corresponding BS equations. Since in this work we only concern the ground states, the orbital angular momentum between two constituent mesons is zero ( $l = 0$ ). For a molecular state whose constituents are one pseudoscalar and one vector, its  $J^{PC}$  is  $1^+$ . For the molecular states which consist of two vector mesons their  $J^{PC}$  may be  $0^+$ ,  $1^+$  and  $2^+$ . We only deduce the BS equation for the bound state with  $J^P = 0^+$  because it should be the lightest molecular state and more favorable in the nature. As a matter of fact, the Lorentz structures of the others are very complicated and will be explored in our later works.

When we solve the BS equations for these molecular states, two approximations are needed: the ladder approximation and instantaneous approximation. In general the energy exchange between the constituents is small compared to  $\Lambda_{QCD}$  which characterizes the binding energy scale of the constituent mesons, so that the instantaneous approximation is applicable. The simple ladder approximation has been employed all along in the history, but recently some works suggest that the cross-ladder should be included[21, 22]. Since our goal is to study the possibility of formation of molecular states instead of making precise theoretical predictions on the spectra we will still use the simple ladder approximation to gain our qualitative conclusion.

After this introduction we derive the BS equations for the  $0^+$  and  $1^+$  molecular states. Then in section III we present our numerical results of the binding energies along with explicitly displaying all input parameters. Section IV is devoted to a brief summary.

## II. THE BETHE-SALPETER FORMALISM

In this section we will deduce the BS equations for the  $0^+$  and  $1^+$  molecular bound states.

### A. The bound state ( $0^+$ ) composed of two pseudoscalar mesons

In Ref. [19, 20] the BS equation of a bound state of two pseudoscalars was deduced. Since in this work we need to use the corresponding formulas for analyzing the spectra of the molecular states containing two pseudoscalars, let us briefly review the main contents of Ref. [19, 20], then in the following subsections, we will step forward to deduce the BS equations for one-vector-one-pseudoscalar bound states and two-vector bound states .

The BS wave function for the bound state  $|\mathcal{P}\rangle$  of two pseudoscalar mesons can be defined as following:

$$\chi_{\mathcal{P}}(x_1, x_2) = \langle 0 | T \phi_1(x_1) \phi_2(x_2) | \mathcal{P} \rangle = e^{-iPX} \chi_{\mathcal{P}}(x), \quad (1)$$

where  $\phi_1(x_1)$  and  $\phi_2(x_2)$  are the field operators of two mesons, respectively,  $P$  denotes the total momentum of the bound state, the relative coordinate  $x$  and the center of mass coordinate  $X$  are

$$X = \eta_1 x_1 + \eta_2 x_2, \quad x = x_1 - x_2, \quad (2)$$

where  $\eta_i = m_i/(m_1 + m_2)$  and  $m_i$  ( $i = 1, 2$ ) is the mass of the  $i$ -th constituent meson. The equation for the BS wave function can be derived from a four-point Green function,

$$S(x_1, x_2; y_2, y_1) = \langle 0 | T \phi_1(x_1) \phi_2(x_2) (\phi_1(y_1) \phi_2(y_2))^\dagger | 0 \rangle. \quad (3)$$

To obtain the BS equation, we express the above four-point Green function in terms of the four-point truncated irreducible kernel  $\overline{K}$ ,

$$S(x_1, x_2; y_2, y_1) = S_{(0)}(x_1, x_2; y_2, y_1) + \int d^4u_1 d^4u_2 d^4v_1 d^4v_2 S_{(0)}(x_1, x_2; u_2, u_1) \overline{K}(u_1, u_2; v_2, v_1) S(v_1, v_2; y_2, y_1), \quad (4)$$

where  $S_{(0)}$  is related to the forward scattering disconnected four-point amplitude,

$$S_{(0)}(x_1, x_2; y_2, y_1) = \Delta_1(x_1, y_1) \Delta_2(x_2, y_2), \quad (5)$$

and  $\Delta_i(x_i, y_i)$  is the complete propagator of the  $i$ -th meson. Here we have

$$\Delta_1^{-1}(p_1, m_1) \Delta_2^{-1}(p_2, m_2) \chi_p(p) = \int \frac{d^4p'}{(2\pi)^4} \overline{K}_1(p, p') \chi_p(p'), \quad (6)$$

where  $\Delta_1 = \frac{i}{p_1^2 - m_1^2}$ ,  $\Delta_2 = \frac{i}{p_2^2 - m_2^2}$  and  $\overline{K}_1(p, p') = -i c_I g^2 \frac{(p_1 + p'_1) \cdot (p_2 + p'_2) + (p_1^2 - p_1'^2)(p_2^2 - p_2'^2)/M_V^2}{(p_1 - p'_1)^2 - M_V^2}$  which has been deduced in Ref. [19, 20].

The relative momenta and the total momentum of the bound state in the equations are defined as

$$p = \eta_2 p_1 - \eta_1 p_2, \quad p' = \eta_2 p'_1 - \eta_1 p'_2, \quad P = p_1 + p_2 = p'_1 + p'_2. \quad (7)$$

Directly solving the BS equation (6) is extremely difficult. In general one needs to use the so-called instantaneous approximation:  $\overline{K}_1(p, p') = K_1(\mathbf{p}, \mathbf{p}')$  by which the BS equation can be reduced to

$$\frac{E^2 - (E_1 + E_2)^2}{(E_1 + E_2)/E_1 E_2} \tilde{\chi}_p(\mathbf{p}) = \frac{i}{2} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} K_{1V(S)}(\mathbf{p}, \mathbf{p}') \tilde{\chi}_p(\mathbf{p}') F(\mathbf{p} - \mathbf{p}')^2, \quad (8)$$

where  $E_i \equiv \sqrt{\mathbf{p}^2 + m_i^2}$ ,  $E = P^0$ , and the equal-time wave function is defined as  $\tilde{\chi}_p(\mathbf{p}) = \int dp^0 \chi_p(p)$ . Since the constituent meson is not a point particle a form factor at each interaction vertex among hadrons must be introduced to reflect the finite-size effects of these hadrons. The form factor is assumed to be in the following form:

$$F(\mathbf{k}) = \frac{2\Lambda^2 - M_V^2}{2\Lambda^2 + \mathbf{k}^2}, \quad \mathbf{k} = \mathbf{p} - \mathbf{p}', \quad (9)$$

where  $\Lambda$  is a cutoff parameter. For exchange of a light vector between the mesons, the kernel is

$$K_{1V}(\mathbf{p}, \mathbf{p}') = i c_I g_{PVP} g'_{PVP} \frac{(\mathbf{p} + \mathbf{p}')^2 + 4\eta_1 \eta_2 E^2 + (\mathbf{p}^2 - \mathbf{p}'^2)^2/M_V^2}{(\mathbf{p} - \mathbf{p}')^2 + M_V^2}. \quad (10)$$

The exchanged mesons between two pseudoscalars are vector mesons, obviously we only need to keep the lightest vector mesons  $\rho$  and  $\omega$  for taking the dominant contributions into account [19, 20]. When the bound state is an isospin-scalar  $c_I = 3$  for  $\rho(\pi)$  and  $c_I = 1$  for  $\omega(\sigma)$ . When the bound state is an isospin-vector  $c_I = -1$  for  $\rho(\pi)$  and  $c_I = 1$  for  $\omega(\sigma)$ .

## B. The bound state( $1^+$ ) composed of a pseudoscalar and a vector

We can define the BS wave function for the bound state  $|\mathcal{V}\rangle$  composed of one pseudoscalar and a vector mesons as following:

$$\chi^\mu_{\mathcal{V}}(x_1, x_2) = \langle 0 | T \phi_1(x_1) \phi_2^\mu(x_2) | \mathcal{V} \rangle = e^{-iPX} \chi^\mu_{\mathcal{V}}(x), \quad (11)$$

where  $\phi_1(x_1)$  and  $\phi_2^\mu(x_2)$  are respectively the field operators of the two mesons. The equation for the BS wave function should be derived from a four-point Green function,

$$S^{\mu\nu}(x_1, x_2; y_2, y_1) = \langle 0 | T \phi_1^\mu(x_1) \phi_2(x_2) (\phi_1^\nu(y_1) \phi_2(y_2))^\dagger | 0 \rangle. \quad (12)$$

To obtain the corresponding BS equation, we write the above four-point Green function in terms of the four-point truncated irreducible kernel  $\bar{K}$ ,

$$S^{\mu\nu}(x_1, x_2; y_2, y_1) = S_{(0)}^{\mu\nu}(x_1, x_2; y_2, y_1) + \int d^4u_1 d^4u_2 d^4v_1 d^4v_2 S_{(0)}^{\mu\alpha}(x_1, x_2; u_2, u_1) \bar{K}_{\alpha\beta}(u_1, u_2; v_2, v_1) S^{\beta\nu}(v_1, v_2; y_2, y_1), \quad (13)$$

where  $S_{(0)}$  is related to the forward-scattering disconnected four-point amplitude,

$$S_{(0)}^{\mu\alpha}(x_1, x_2; y_2, y_1) = \Delta_1(x_1, y_1) \Delta_2^{\mu\alpha}(x_2, y_2), \quad (14)$$

and  $\Delta_i(x_i, y_i)$  is the full propagator of the  $i$ -th particle,

$$\Delta_1^{-1}(p_1, m_1) \Delta_{2\mu\alpha}^{-1}(p_2, m_2) \chi^\mu_{\mathcal{V}}(p) = \int \frac{d^4p'}{(2\pi)^4} \bar{K}_{\alpha\beta}(p, p') \chi^\beta_{\mathcal{V}}(p'). \quad (15)$$

Here  $\Delta_1 = \frac{i}{p_1^2 - m_1^2}$  and  $\Delta_{2\mu\alpha} = \frac{i}{p_2^2 - m_2^2} (\frac{p_{2\mu} p_{2\alpha}}{m_2^2} - g_{\mu\alpha})$  are the propagators of pseudoscalar and vector mesons.

From Eq. (15) one can obtain

$$\frac{-1}{(p_1^2 - m_1^2)(p_2^2 - m_2^2)} \chi^\mu_{\mathcal{V}}(p) = \int \frac{d^4p'}{(2\pi)^4} \bar{K}_{\alpha\beta}(p, p') \chi^\beta_{\mathcal{V}}(p') (\frac{p_2^\mu p_2^\alpha}{m_2^2} - g^{\mu\alpha}). \quad (16)$$

We write  $\chi^\mu_{\mathcal{V}}(p) = \chi_{\mathcal{V}}(p) \epsilon^\mu$ , then multiply an  $\epsilon_\mu^*$  on both sides. Summing over the polarizations we deduce a new equation

$$\frac{-1}{(p_1^2 - m_1^2)(p_2^2 - m_2^2)} \chi_{\mathcal{V}}(p) = \int \frac{d^4p'}{(2\pi)^4} \bar{K}_{\alpha\beta}(p, p') \chi_{\mathcal{V}}(p') (\frac{p_2^\mu p_2^\alpha}{m_2^2} - g^{\mu\alpha}) (\frac{p'_\mu p'^\beta}{p'^2} - g_\mu^\beta). \quad (17)$$

With the Feynman diagrams depicted in Fig.1 and the effective interactions[14, 19] we eventually obtain

$$\bar{K}_{\alpha\beta} = g_{PVP} g_{V\bar{V}V} \frac{\frac{q^\lambda q^\sigma}{q^2} - g^{\lambda\sigma}}{q^2 - M_V^2} (p_1 + p'_1)_\lambda [-g_{\alpha\beta} (p_2 + p'_2)_\sigma + p'_{2\alpha} g_{\sigma\beta} + p_{2\beta} g_{\sigma\alpha}].$$

Defining  $\overline{K}_2(p, p') = \overline{K}_{\alpha\beta}(p, p')(\frac{p_2^\mu p_2^\alpha}{m_2^2} - g^{\mu\alpha})(\frac{p_1'^\mu p_1'^\beta}{p'^2} - g_\mu^\beta)$  and employing the instantaneous approximation we obtain the BS equation similar to Eq. (6) and (8) but with a different kernel,

$$\frac{E^2 - (E_1 + E_2)^2}{(E_1 + E_2)/E_1 E_2} \tilde{\chi}_\nu(\mathbf{p}) = \frac{i}{2} \int \frac{d^3 \mathbf{p}'}{(2\pi)^3} K_{2V(S)}(\mathbf{p}, \mathbf{p}') \tilde{\chi}_\nu(\mathbf{p}') F(\mathbf{p} - \mathbf{p}')^2. \quad (18)$$

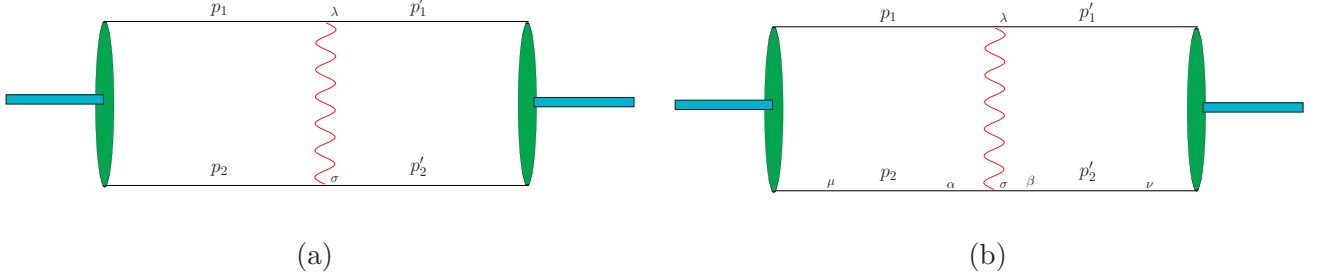


FIG. 1: (a) A bound state composed of two pseudoscalars (b) a bound state composed of a pseudoscalar and a vector. In the two Feynman diagrams  $\rho$  and  $\omega$  are exchanged.

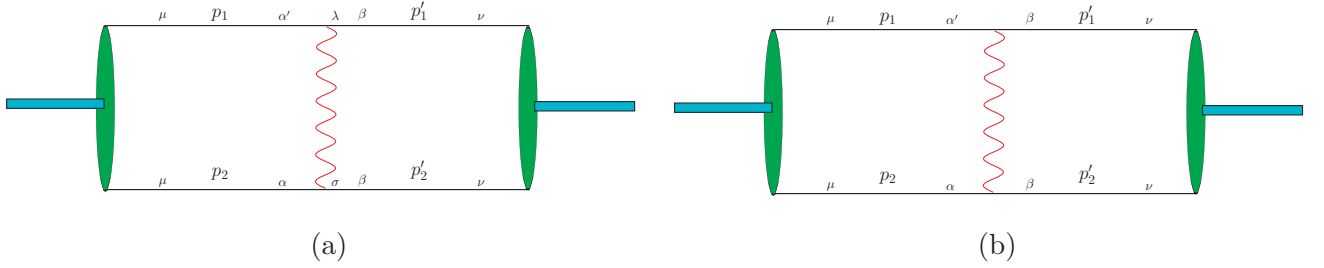


FIG. 2: A bound state composed of two vectors. (a)  $\rho$  and  $\omega$  are exchanged (b)  $\pi$  is exchanged

### C. The bound state( $0^+$ ) composed of two vector mesons

The quantum number  $J^P$  of the bound state composed of two vectors can be  $0^+$ ,  $1^+$  and  $2^+$ . As aforementioned, since  $0^+$  is more favorable in nature and its Lorentz structure is relatively simpler than the two others, in this work we only study the  $0^+$  bound states and define the corresponding BS wave function  $|\mathcal{S}\rangle$  as following:

$$\chi_{\mathcal{S}}(x_1, x_2) = \langle 0 | T \phi_{1\mu}(x_1) \phi_2^\mu(x_2) | \mathcal{S} \rangle = e^{-iPX} \chi_{\mathcal{S}}(x). \quad (19)$$

The equation for the BS wave function can be derived from a four-point Green function,

$$S(x_1, x_2; y_2, y_1) = \langle 0 | T \phi_{1\mu}(x_1) \phi_2^\mu(x_2) (\phi_{1\nu}(y_1) \phi_2^\nu(y_2))^\dagger | 0 \rangle. \quad (20)$$

In analog to the procedures used in last subsection, we obtain

$$\Delta_{1\mu\alpha}^{-1}(p_1, m_1)\Delta_2^{-1\mu\alpha'}(p_2, m_2)\chi_s(p) = \int \frac{d^4p'}{(2\pi)^4} \bar{K}_\alpha^{\alpha'}(p, p')\chi_s(p'), \quad (21)$$

where  $\Delta_{1\mu\alpha} = \frac{i}{p_1^2 - m_1^2}(\frac{p_{1\mu}p_{1\alpha}}{m_1^2} - g_{\mu\alpha})$  and  $\Delta_{2\mu\alpha'} = \frac{i}{p_2^2 - m_2^2}(\frac{p_{2\mu}p_{2\alpha'}}{m_2^2} - g_{\mu\alpha'})$ .

From Eq. (15) one can obtain

$$\frac{-1}{(p_1^2 - m_1^2)(p_2^2 - m_2^2)}\chi_s^\mu(p) = \int \frac{d^4p'}{(2\pi)^4} \bar{K}_\alpha^{\alpha'}(p, p')\chi_s(p')(\frac{p_2^\mu p_2^{\alpha'}}{m_2^2} - g^{\mu\alpha})(\frac{p_{1\mu}p_{1\alpha'}}{m_2^2} - g_{\mu\alpha'}). \quad (22)$$

With the Feynman diagrams depicted in Fig.2 and the effective interaction[19, 20, 23, 24] we can obtain

$$\bar{K}_{V\alpha}^{\alpha'} = g_{VVV}g'_{VVV}\frac{q^\lambda q^\sigma - g^{\lambda\sigma}}{q^2 - M_V^2}[-g^{\alpha'\beta}(p_1+p'_1)_\lambda + p_1^{\alpha'}g_\lambda^\beta + p_1^\beta g_\lambda^{\alpha'}][-g_{\alpha\beta}(p_2+p'_2)_\sigma + p_{2\alpha}'g_{\sigma\beta} + p_{2\beta}'g_{\sigma\alpha}]$$

and

$$\bar{K}_{P\alpha}^{\alpha'} = g_{VPV}g'_{VPV}\varepsilon^{\alpha'\beta\sigma\rho}q_\sigma p_{1\rho}\varepsilon_{\alpha\beta\sigma'\rho'}q^{\sigma'}p_2^{\rho'}.$$

Defining  $\bar{K}_{3V(P,S)}(p, p') = \bar{K}_{V(P,S)\alpha}^{\alpha'}(p, p')(\frac{p_2^\mu p_2^{\alpha'}}{m_2^2} - g^{\mu\alpha})(\frac{p_{1\mu}p_{1\alpha'}}{m_2^2} - g_{\mu\alpha'})$  we derive the BS equation which is similar to Eqs. (6) and (8) but possesses a different kernel,

$$\frac{E^2 - (E_1 + E_2)^2}{(E_1 + E_2)/E_1 E_2}\tilde{\chi}_s(\mathbf{p}) = \frac{i}{2} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} K_{3V(P,S)}(\mathbf{p}, \mathbf{p}')\tilde{\chi}_s(\mathbf{p}')F(\mathbf{p} - \mathbf{p}')^2. \quad (23)$$

### III. NUMERICAL RESULTS

Solving the BS equations Eq. (8), (18) and (23), we obtain the eigenvalues for each state. To determine whether such a state composed of two heavy mesons is a bound state, a criterion must be set. That is: if the eigenvalue of the bound state obtained by solving the BS equation is negative, namely the total mass of the system is lower than the sum of the masses of the two constituents, such a system is considered as a bound state, i.e. a hadronic molecule which may exist in the nature.

Now let us solve the BS equations. Since the function  $\tilde{\chi}_P(\mathbf{p})$ ,  $\tilde{\chi}_V(\mathbf{p})$ ,  $\tilde{\chi}_S(\mathbf{p})$  only depends on the norm of the three-momentum we may first integrate over the azimuthal angle of the function in (8), (18) or (23)

$$\frac{i}{2} \int \frac{d^3\mathbf{p}'}{(2\pi)^3} K(\mathbf{p}, \mathbf{p}')F(\mathbf{p} - \mathbf{p}')^2,$$

to obtain a new form  $U(|\mathbf{p}|, |\mathbf{p}'|)$ , then the BS equation turns into a one-dimension integral equation

$$\tilde{\chi}(|\mathbf{p}|) - \frac{(E_1 + E_2)/E_1 E_2}{E^2 - (E_1 + E_2)^2} \int d|\mathbf{p}'| U(|\mathbf{p}|, |\mathbf{p}'|)\tilde{\chi}(|\mathbf{p}'|) = 0. \quad (24)$$

Letting  $|\mathbf{p}|(|\mathbf{p}'|)$  take ordered  $n$  discrete values and the gap between two adjacent values be  $\Delta\mathbf{p}$ , then  $\chi(|\mathbf{p}|)$  can be arranged as a column matrix and the coefficients constitute an  $n \times n$  matrix  $M$ . The scalar equation of  $M$  would determine the  $n$  eigenvalues  $E_n$  and the smallest one should correspond to the binding energy of the ground state. In our calculation we take  $n = 151$  and the first discrete value of  $|\mathbf{p}| = 0.001$  GeV and the largest  $|\mathbf{p}|$  is 2 GeV.

In our calculation we need to determine the values of the parameters  $\Lambda$ ,  $g_{PVP}$ ,  $g_{VVV}$ ,  $g_{PSP}$  and  $g_{VSV}$ . By fitting the data of the decay processes where the couplings  $KK\rho$  and  $BB\rho$  are concerned, the authors of Ref.[20] obtained the cutoff parameter  $\Lambda$  which can vary from 1GeV to 4GeV. Their strategy was to fit the measured decay widths in terms of the theoretically derived formulas with  $\Lambda$ , thus they obtained the rather wide range for  $\Lambda$ . Indeed such a wide range reduces our theoretical prediction power. Instead, in Ref. [25] the authors suggested a relation:  $\Lambda = m + \alpha\Lambda_{QCD}$  where  $m$  is the mass of the exchanged meson,  $\alpha$  is a number of  $O(1)$  and  $\Lambda_{QCD} = 220$  MeV i.e.  $\Lambda \sim 1\text{GeV}$  for exchanging  $\rho$  or  $\omega$ . Since there are uncertainties on  $\Lambda$  we will choose  $\Lambda = 1.5$  GeV,  $\Lambda = 2.5$  GeV and  $\Lambda = 3.5$  GeV to calculate the eigenvalues and compare the results. The coupling constants  $g_{BB\rho} = g_{KK\rho} = 3$  are fixed by fitting data [23]. Considering the flavor  $SU(3)$  symmetry and heavy quark spin symmetry we believe that  $g_{BB\rho(\omega)} = g_{DD\rho(\omega)} = g_{BB^*\rho(\omega)} = g_{DD^*\rho(\omega)} = g_{B^*B^*\rho(\omega)} = g_{D^*D^*\rho(\omega)} = g$  should be a not-bad approximation. Instead of setting  $g = 3$ , we also let  $g$  take various values from 2 to 4 to investigate dependence of the eigenvalues on those coupling constants.

The masses of the concerned constituent mesons are directly taken from the databook [26] as:  $m_D = 1.865$  GeV,  $m_{D^*} = 2.007$  GeV,  $m_B = 5.279$  GeV and  $m_{B^*} = 5.325$  GeV .

### A. The exchanged mesons are $\rho$ and $\omega$

It is noted that in those tables there are many places symbolized by “—” which means such bound states cannot exist because either the BS equation with the corresponding parameters has no solution at all, or the derived eigen-energy is not negative, namely the mass of the supposed-to-be bound state is larger than the sum of the two constituent mesons.

We calculate the eigenvalues of these bound states of  $D\bar{D}(0^+)$ ,  $D\bar{D}^*(1^+)$ ,  $D^*\bar{D}^*(0^+)$ ,  $B\bar{B}(0^+)$ ,  $B\bar{B}^*(1^+)$ ,  $B^*\bar{B}^*(0^+)$  respectively. Apparently when the parameters  $\Lambda$  and  $g_{PVP}(g_{VVV})$  are reasonable, the corresponding BS equation is solvable. For the bound states of isospin  $I = 0$  we can obtain the ground eigenvalues (see Tab. I). It implies that the two heavy mesons can form an isospin-0 molecular state by exchanging light mesons. However for  $I = 1$  we cannot obtain eigenvalues which are below the threshold with the same parameters. The reason is that the contributions of exchanging  $\rho$  and  $\omega$  nearly cancel each other in the kernel  $K$ .



TABLE I: The eigenvalues of the ground molecular states ( $I = 0$ ) when only  $\rho$  and  $\omega$  are exchanged between the constituents

	$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}^*$	$B\bar{B}$	$B\bar{B}^*$	$B^*\bar{B}^*$
$\Lambda = 1.5^a$	-	3.877	3.406	10.50	10.54	9.904
$\Lambda = 2.5^a$	3.708	3.844	2.699	10.38	10.43	9.245
$\Lambda = 3.5^a$	3.680	3.815	2.350	10.33	10.37	8.950
$\Lambda = 1.5^b$	3.618	3.756	3.892	10.26	10.31	8.899
$\Lambda = 2.5^b$	3.379	3.529	3.845	9.955	10.00	7.771
$\Lambda = 3.5^b$	3.254	3.414	3.841	9.800	9.857	7.283
$\Lambda = 1.5^c$	3.330	3.480	3.637	9.909	9.956	7.798
$\Lambda = 2.5^c$	2.860	3.067	3.878	9.392	9.446	6.356
$\Lambda = 3.5^c$	2.656	2.886	3.935	9.162	9.219	5.802

<sup>a</sup>the coupling constants  $g_{PVP(VVV)}$  are set to be 2

<sup>b</sup>the coupling constants  $g_{PVP(VVV)}$  are set to be 3

<sup>c</sup>the coupling constants  $g_{PVP(VVV)}$  are set to be 4

TABLE II: The eigenvalues of the ground molecular states when the contribution of  $\pi$  are included with  $g_{D^*\bar{D}^*\pi} = g_{B^*\bar{B}^*\pi} = 18$  and the coupling constants  $g_{PVP(VVV)}$  are set to be 3.

	$D^*\bar{D}^*(I=0)$	$B^*\bar{B}^*(I=0)$	$D^*\bar{D}^*(I=1)$	$B^*\bar{B}^*(I=1)$
$\Lambda = 1.5$	3.588	8.489	-	-
$\Lambda = 2.5$	3.606	7.270	-	-
$\Lambda = 3.5$	3.640	6.760	-	-

## B. As exchange of $\pi$ between two vectors is taken into account

For the bound states composed of two vectors such as  $B^*\bar{B}^*$  and  $D^*\bar{D}^*$ , the contribution of pion-exchange should be included. Including  $\pi$ -exchange in the kernel, we repeat our derivation and obtain a new BS equation. Using the data of  $D^{*+} \rightarrow D^0\pi^+$  [26] and the formula given in Ref. [27] we fix  $g_{D^*\bar{D}^*\pi} = 18$ , and furthermore under the heavy quark symmetry we may set  $g_{B^*\bar{B}^*\pi} = g_{D^*\bar{D}^*\pi} = g_{D^*\bar{D}^*\pi}$ . With this coupling constant we calculate the eigenvalues of the bound states  $D^*\bar{D}^*$  and  $B^*\bar{B}^*$  and the results are listed in Tab.II. Comparing the values in Tab.II with that in Tab.I we find that the masses of the molecular states  $D^*\bar{D}^*$  and  $B^*\bar{B}^*(I=0)$  slightly decrease by less than 10% as the pion-exchange between the two vectors is taken into account. It shows that the pion-exchange contributes a weak attractive effect, but it is not as important as that from the vector exchange. Moreover, for the  $I=1$  bound state the pion-exchange with  $C_I = -1$  is even less important so there is still no  $I=1$  bound state as indicated by our numerical results.

TABLE III: The masses of the ground molecular states when one set  $\Lambda = m + \alpha\Lambda_{QCD}$  with  $g_{B\bar{B}\rho} = g_{B\bar{B}\omega} = 3$  and  $g_{D^*\bar{D}^*\pi} = g_{B^*\bar{B}^*\pi}$

	$B\bar{B}$ ( $I = 0$ )	$B\bar{B}^*$ ( $I = 0$ )	$B^*\bar{B}^*$ <sup>a</sup> ( $I = 0$ )	$B\bar{B}$ ( $I = 1$ )	$B\bar{B}^*$ ( $I = 1$ )	$B^*\bar{B}^*$ ( $I = 1$ )
$\alpha = 1$	10.49	10.53	9.939	-	-	-
$\alpha = 2$	10.39	10.44	9.364	-	-	-

<sup>a</sup>the contribution of exchanged  $\pi$  has been included.

TABLE IV: The masses of the ground molecular states and the coupling constants  $g_{PVP(VVV)}$  are set to be 3 when the contribution of  $\sigma$  are included.

	$D\bar{D}$	$D\bar{D}^*$	$D^*\bar{D}^*$ <sup>a</sup>	$B\bar{B}$	$B\bar{B}^*$	$B^*\bar{B}^*$
$\Lambda = 2.5, g_\sigma = 0.76$ ( $I = 0$ )	3.386	3.538	3.839	9.967	9.990	7.750
$\Lambda = 2.5, g_\sigma = 0.76$ ( $I = 1$ )	-	-	-	-	-	-
$\Lambda = 2.5, g_\sigma = 3$ ( $I = 0$ )	3.505	3.382	3.750	10.15	9.801	7.130
$\Lambda = 2.5, g_\sigma = 3$ ( $I = 1$ )	-	-	3.760	-	10.56	10.22

<sup>a</sup>the contribution of exchanging  $\pi$  is not included.

As aforementioned one also can set  $\Lambda = m + \alpha\Lambda_{QCD}$  which would determine values of  $\Lambda$  lower than that we have used and the relation implies different  $\Lambda$  values for different exchanged mesons. We calculate the masses of the  $B^{(*)}\bar{B}^{(*)}$  system for  $\alpha = 1$  and  $\alpha = 2$  respectively and the numerical results are presented in Tab.III. The results in Tab.III are qualitatively consistent with the conclusions we have made.

### C. As exchange of $\sigma$ is taken into account

Moreover, two pseudoscalars and two vectors can couple to a scalar, so that one should include the contribution of  $\sigma$ -exchange in the kernel. In terms of the effective Lagrangian[28] we deduce the corresponding kernel (See appendix). With the contribution of  $\sigma$  the eigenvalues of the bound state  $D\bar{D}(0^+)$ ,  $D\bar{D}^*(1^+)$ ,  $D^*\bar{D}^*(0^+)$ ,  $B\bar{B}(0^+)$ ,  $B\bar{B}^*(1^+)$ ,  $B^*\bar{B}^*(0^+)$  are re-calculated. The parameter  $g_\sigma$  is determined by the heavy hadron chiral perturbation theory (HHChPT) as  $g_\sigma = \frac{g_\pi}{2\sqrt{6}} \approx 0.76$  with  $g_\pi = 3.73^1$  [28]. Using this value of  $g_\sigma$  we repeat our computations and the resultant masses of the ground molecular states are listed in Tab.IV. Comparing Tab.I with Tab.IV we find the contribution of  $\sigma$  is very small for the concerned case.

<sup>1</sup> We thank Dr. Valery E. Lyubovitskij for explaining how to determine the value of  $g_\pi$ .

The above numerical results show that one cannot obtain an  $I = 1$  bound state which is composed of two heavy mesons. The reason is the contributions from exchanges of  $\rho$  and  $\omega$  cancel each other for the quantum number  $I = 1$ , and the small coupling constant  $g_\sigma$  determines that including the  $\sigma$  contribution cannot make a substantial change to the kernel. Let us take another angle to look at this issue. With the understanding, now let us deliberately enhance the contribution of  $\sigma$  by enlarging its coupling with pseudoscalar and vector mesons. Namely, we investigate to what value the coupling constant  $g_\sigma$  reaches, the two heavy mesons can form a molecular state. We notice that as the coupling constant  $g_\sigma = 3$  the eigenvalues for  $B\bar{B}^*(I = 1, J^P = 1^+)$ ,  $B^*\bar{B}^*(I = 1, J^P = 0^+)$  and  $D^*\bar{D}^*(I = 1, J^P = 0^+)$  meet the criterion for forming stable molecular states and the results are shown in Tab.IV. Moreover, as we set the coupling constant  $g_\sigma = 4$  and  $\Lambda = 2.5$  we can also obtain the mass of 3.864 GeV for  $D\bar{D}^*(I = 1, J^P = 1^+)$ . It is worth emphasizing that when we set  $\Lambda = 2.5$  GeV and  $g_\sigma = 3$  the mass of the bound state  $B\bar{B}^*(I = 1, J^P = 1^+)$  is 10.56 GeV which is close to the mass of the newly observed narrow-structure resonance  $Z_b(10610)$   $[(10608.4 \pm 2.0) \text{ MeV}]$ .

#### IV. A BRIEF SUMMARY

In this work we study the possibility of two heavy mesons forming a hadronic molecule. We employ the BS framework to approach this goal because it may include the relativistic corrections automatically. In Ref. [20] the BS equation for the bound state of two pseudoscalar mesons was deduced whereas in this work we derive the BS equations of the bound states composed of one vector and one pseudoscalar with  $J^P = 1^+$  and the bound states composed of two vectors with  $J^P = 0^+$ . Our numerical results indicate that when the parameters are within reasonable ranges two mesons indeed can form an  $I = 0$  molecular state but cannot form an  $I = 1$  molecular state because the contribution of  $\sigma$ -meson is small, thus we conclude that the newly observed  $Z_b(10610)$  prefers a tetraquark structure or at most has a tiny fraction of the molecular state of  $B\bar{B}^*$ . By contrast, if the coupling constant  $g_\sigma$  is enhanced by several times an  $I = 1$  molecular state might be formed. For example when we set  $\Lambda = 2.5$  GeV and  $g_\sigma = 3$  the mass of the  $I = 1$  bound state  $B\bar{B}^*(1^+)$  is 10.56 GeV which is close to the mass of the newly observed narrow-structure resonance  $Z_b(10610)$ .

Therefore our qualitative conclusion is that either the observed  $Z_b(10610)$  is not a molecular state or the coupling of  $\sigma$ -meson with the pseudoscalar and vector mesons is several times larger than that given in the HHChPT.

Since the parameters are fixed in various experiments and can span a relatively large range we cannot expect all the numerical results which depend on the parameters, to be very accurate. The goal of this work is to study the possibility of two heavy mesons can form a molecular state. Our results, even not accurate, have obvious qualitative significance.

In a recent work, Ali et al. [30] propose the tetraquark interpretation of the charged

bottomonium-like states  $Z_b^\pm(10610)$  and  $Z_b^\pm(10650)$ , and find that with this ansatz their estimates on the production and decay fit the data well, thus they claim that the tetraquark interpretation is supported by the experimental measurements. This is consistent with our conclusion that the tetraquark structure of  $Z_b^\pm(10610)$  is preferred. Definitely, further theoretical and experimental works are badly needed for gaining better understanding of the exotic structures of the heavy mesons.

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## Appendix A: Notations

The effective interactions are [20, 28, 29]

$$\begin{aligned}
\mathcal{L}_{MM\rho} &= ig_{MM\rho} [\bar{M} \vec{\tau} (\partial_\mu M) - (\partial_\mu \bar{M}) \vec{\tau} M] \cdot \vec{\rho}^\mu \\
\mathcal{L}_{MM\omega} &= ig_{MM\omega} [\bar{M} (\partial_\mu M) - (\partial_\mu \bar{M}) M] \cdot \omega^\mu \\
\mathcal{L}_{\rho M^* M^*} &= ig_{\rho M^* M^*} \vec{\rho}^\mu \cdot [-M^{*\nu} \vec{\tau} (\vec{\partial}_\mu - \overleftarrow{\partial}_\mu) M_\nu^{*\dagger} + M^{*\nu} \vec{\tau} \partial_\nu M_\mu^{*\dagger} - \partial^\nu M_{*\mu} \vec{\tau} M_{*\nu}^\dagger], \\
\mathcal{L}_{\omega M^* M^*} &= ig_{\omega M^* M^*} [-\omega^\mu M^{*\nu} (\vec{\partial}_\mu - \overleftarrow{\partial}_\mu) M_\nu^{*\dagger} + \omega^\mu M^{*\nu} \partial_\nu M_\mu^{*\dagger} - \omega_\mu \partial_\nu M^{*\mu} M^{*\nu\dagger}], \\
\mathcal{L}_{M^* M^* \pi} &= \frac{g_{M^* M^* \pi}}{m_{M^*}} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu M_\nu^* \vec{\tau} M_\alpha^\dagger \cdot \partial_\beta \vec{\pi} \\
\mathcal{L}_{M^* M^* \sigma} &= 2m_{M^*} g_\sigma M^* \cdot M^{*\dagger} \sigma \\
\mathcal{L}_{MM\sigma} &= -2m_M g_\sigma M \cdot M^\dagger \sigma
\end{aligned} \tag{A1}$$

and kernel

$$\begin{aligned}
K_{2V}(\mathbf{p}, \mathbf{p}') &= \frac{ic_I g_1 g_1'}{3(-M_V^2 + q^2)} [4\mathbf{p} \cdot \mathbf{p}' - \frac{\mathbf{p} \cdot \mathbf{p}'^2}{m_2^2} + 2\mathbf{p}'^2 - \frac{\mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}'^2}{m_2^2 M_V^2} + \frac{2\mathbf{p}'^2{}^2}{M_V^2} + 2\mathbf{p}^2 \\
&+ \frac{\mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}^2}{m_2^2 M_V^2} + \frac{\mathbf{p}'^2 \mathbf{p}^2}{m_2^2} - \frac{4\mathbf{p}'^2 \mathbf{p}^2}{M_V^2} + \frac{\mathbf{p}'^2{}^2 \mathbf{p}^2}{m_2^2 M_V^2} + \frac{2\mathbf{p}^2{}^2}{M_V^2} - \frac{\mathbf{p}'^2 \mathbf{p}^2{}^2}{m_2^2 M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \eta_2^2}{m_2^2} \\
&+ \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \eta_2^2}{m_2^2 M_V^2} + \frac{E^2 \mathbf{p}^2 \eta_2^2}{m_2^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_2^2}{m_2^2 M_V^2} - \frac{E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_2^2}{m_2^2 M_V^2} \\
&+ \frac{E^2 \mathbf{p}^2{}^2 \eta_2^2}{m_2^2 M_V^2} + 2\eta_1 (6E^2 \eta_2 + \frac{E^2 \mathbf{p}^2 \eta_2}{m_2^2})],
\end{aligned} \tag{A2}$$

$$K_{3P}(\mathbf{p}, \mathbf{p}') = \frac{2c_I g_2 g'_2}{m_P^2 - q^2} \left[ \mathbf{p} \cdot \mathbf{p}'^2 + 2 E^2 \mathbf{p} \cdot \mathbf{p}' \eta_1 \eta_2 - E^2 \mathbf{p}'^2 \eta_1 \eta_2 - \mathbf{p}^2 (\mathbf{p}'^2 + E^2 \eta_1 \eta_2) \right] \quad (\text{A3})$$

$$\begin{aligned} K_{3V}(\mathbf{p}, \mathbf{p}') = & \frac{ic_I g_3 g'_3}{-M_V^2 + q^2} \left[ -6 \mathbf{p} \cdot \mathbf{p}' - \frac{2 \mathbf{p} \cdot \mathbf{p}'^2}{m_1^2} - \frac{2 \mathbf{p} \cdot \mathbf{p}'^2}{m_2^2} + \frac{2 \mathbf{p} \cdot \mathbf{p}'^2}{M_V^2} - 6 \mathbf{p}'^2 + \frac{\mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}'^2}{m_1^2 M_V^2} \right. \\ & + \frac{\mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}'^2}{m_2^2 M_V^2} - \frac{3 \mathbf{p}'^{22}}{M_V^2} - 6 \mathbf{p}^2 - \frac{2 \mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}^2}{m_1^2 m_2^2} - \frac{\mathbf{p}'^2 \mathbf{p}^2}{m_1^2} - \frac{\mathbf{p}'^2 \mathbf{p}^2}{m_2^2} + \frac{4 \mathbf{p}'^2 \mathbf{p}^2}{M_V^2} + \\ & \frac{\mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}'^2 \mathbf{p}^2}{m_1^2 m_2^2 M_V^2} - \frac{\mathbf{p}'^{22} \mathbf{p}^2}{m_1^2 M_V^2} - \frac{\mathbf{p}'^{22} \mathbf{p}^2}{m_2^2 M_V^2} - \frac{3 \mathbf{p}^{22}}{M_V^2} - \frac{\mathbf{p}'^2 \mathbf{p}^{22}}{m_1^2 m_2^2} - \frac{\mathbf{p}'^{22} \mathbf{p}^{22}}{m_1^2 m_2^2 M_V^2} + \\ & \frac{2 E^2 \mathbf{p} \cdot \mathbf{p}' \eta_1^2}{m_1^2} + \frac{E^2 \mathbf{p}'^2 \eta_1^2}{m_1^2} + \frac{E^2 \mathbf{p}'^{22} \eta_1^2}{m_1^2 M_V^2} + \frac{2 E^2 \mathbf{p}^2 \eta_1^2}{m_1^2} - \frac{2 E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_1^2}{m_1^2 M_V^2} \\ & + \frac{E^2 \mathbf{p}^{22} \eta_1^2}{m_1^2 M_V^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \eta_1 \eta_2}{m_1^2} - \frac{2 E^2 \mathbf{p} \cdot \mathbf{p}'^2 \eta_1 \eta_2}{m_1^2 m_2^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}'^2 \eta_1 \eta_2}{m_1^2 M_V^2} \\ & + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \eta_1 \eta_2}{m_1^2 M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}'^2 \eta_1 \eta_2}{m_1^2 m_2^2 M_V^2} + \frac{3 E^2 \mathbf{p}^2 \eta_1 \eta_2}{m_1^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1 \eta_2}{m_1^2 m_2^2} \\ & - \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1 \eta_2}{m_1^2 M_V^2} - \frac{2 E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_1 \eta_2}{m_1^2 m_2^2} - \frac{2 E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_1 \eta_2}{m_1^2 M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \mathbf{p}^2 \eta_1 \eta_2}{m_1^2 m_2^2 M_V^2} \\ & - \frac{2 E^2 \mathbf{p}'^{22} \mathbf{p}^2 \eta_1 \eta_2}{m_1^2 m_2^2 M_V^2} + \frac{E^2 \mathbf{p}^{22} \eta_1 \eta_2}{m_1^2 m_2^2} + \frac{E^2 \mathbf{p}^{22} \eta_1 \eta_2}{m_1^2 M_V^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^{22} \eta_1 \eta_2}{m_1^2 m_2^2 M_V^2} \\ & + \frac{E^2 \mathbf{p}'^2 \mathbf{p}^{22} \eta_1 \eta_2}{m_1^2 m_2^2 M_V^2} + \frac{3 E^2 \mathbf{p} \cdot \mathbf{p}' \eta_2^2}{m_2^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}'^2 \eta_2^2}{m_2^2 M_V^2} + \frac{E^2 \mathbf{p}'^2 \eta_2^2}{m_2^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \eta_2^2}{m_2^2 M_V^2} \\ & + \frac{E^2 \mathbf{p}'^{22} \eta_2^2}{m_2^2 M_V^2} - \frac{E^2 \mathbf{p}^2 \eta_2^2}{m_2^2} + \frac{2 E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_2^2}{m_1^2 m_2^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_2^2}{m_2^2 M_V^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}^2 \eta_2^2}{m_1^2 m_2^2 M_V^2} \\ & + \frac{E^2 \mathbf{p}'^2 \mathbf{p}^{22} \eta_2^2}{m_1^2 m_2^2 M_V^2} - \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \eta_1^2 \eta_2^2}{m_1^2 m_2^2} - \frac{E^4 \mathbf{p}'^2 \eta_1^2 \eta_2^2}{m_1^2 m_2^2} + \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \eta_1^2 \eta_2^2}{m_1^2 m_2^2 M_V^2} \\ & - \frac{E^4 \mathbf{p}'^{22} \eta_1^2 \eta_2^2}{m_1^2 m_2^2 M_V^2} + \frac{E^4 \mathbf{p}^2 \eta_1^2 \eta_2^2}{m_1^2 m_2^2} - \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1^2 \eta_2^2}{m_1^2 m_2^2 M_V^2} + \frac{E^4 \mathbf{p}'^2 \mathbf{p}^2 \eta_1^2 \eta_2^2}{m_1^2 m_2^2 M_V^2} \\ & + \frac{2 E^4 \mathbf{p} \cdot \mathbf{p}' \eta_1 \eta_2^3}{m_1^2 m_2^2} - \frac{E^4 \mathbf{p} \cdot \mathbf{p}'^2 \eta_1 \eta_2^3}{m_1^2 m_2^2 M_V^2} + \frac{E^4 \mathbf{p}'^2 \mathbf{p}^2 \eta_1 \eta_2^3}{m_1^2 m_2^2 M_V^2} - 2 \eta_1 \left( \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \eta_1}{m_1^2} \right. \\ & + \frac{E^2 \mathbf{p}^2 \eta_1}{m_1^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1}{m_1^2 m_2^2} + 6 E^2 \eta_2 - \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \eta_2}{m_2^2} + \frac{2 E^2 \mathbf{p}^2 \eta_2}{m_1^2} + \frac{E^2 \mathbf{p}^2 \eta_2}{m_2^2} \\ & + \frac{E^2 \mathbf{p}^{22} \eta_2}{m_1^2 m_2^2} + \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \eta_1^2 \eta_2}{m_1^2 m_2^2} + \frac{E^4 \mathbf{p}^2 \eta_1 \eta_2^2}{m_1^2 m_2^2} \left. \right) + \eta_2 \left[ -\frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \eta_1}{m_1^2 M_V^2} - \frac{E^2 \mathbf{p}^2 \eta_1}{m_1^2} \right. \\ & + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1}{m_1^2 m_2^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1}{m_1^2 M_V^2} + \frac{E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_1}{m_1^2 M_V^2} - \frac{E^2 \mathbf{p}^{22} \eta_1}{m_1^2 M_V^2} - 3 E^2 \eta_2 \\ & \left. + \frac{2 E^2 \mathbf{p} \cdot \mathbf{p}' \eta_2}{M_V^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}'^2 \eta_2}{m_1^2 M_V^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}'^2 \eta_2}{m_2^2 M_V^2} - \frac{E^2 \mathbf{p}'^2 \eta_2}{M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \eta_2}{m_2^2 M_V^2} \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{E^2 \mathbf{p}^2 \eta_2}{m_1^2} - \frac{E^2 \mathbf{p}^2 \eta_2}{M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_2}{m_1^2 m_2^2} + \frac{2 E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_2}{m_1^2 M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_2}{m_2^2 M_V^2} - \\
& \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_2}{m_1^2 m_2^2 M_V^2} - \frac{E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_2}{m_2^2 M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \mathbf{p}^2 \eta_2}{m_1^2 m_2^2 M_V^2} - \frac{E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_2}{m_1^2 M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \eta_2}{m_1^2 m_2^2 M_V^2} \\
& - \frac{E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_2}{m_1^2 m_2^2 M_V^2} + \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \eta_1^2 \eta_2}{m_1^2 m_2^2} + \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \eta_1 \eta_2^2}{m_1^2 m_2^2} - \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \eta_1 \eta_2^2}{m_1^2 m_2^2 M_V^2} \\
& + \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}'^2 \eta_1 \eta_2^2}{m_1^2 m_2^2 M_V^2} + \frac{E^4 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1 \eta_2^2}{m_1^2 m_2^2 M_V^2} - \frac{E^4 \mathbf{p}'^2 \mathbf{p}^2 \eta_1 \eta_2^2}{m_1^2 m_2^2 M_V^2} + \eta_1 \left( \frac{3 E^2 \mathbf{p} \cdot \mathbf{p}' \eta_1}{m_1^2} \right. \\
& - \frac{E^2 \mathbf{p} \cdot \mathbf{p}'^2 \eta_1}{m_1^2 M_V^2} + \frac{3 E^2 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1}{m_1^2 m_2^2} - \frac{E^2 \mathbf{p} \cdot \mathbf{p}'^2 \mathbf{p}^2 \eta_1}{m_1^2 m_2^2 M_V^2} + \frac{E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_1}{m_1^2 M_V^2} \\
& + \frac{E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_1}{m_1^2 m_2^2 M_V^2} - 3 E^2 \eta_2 - \frac{3 E^2 \mathbf{p} \cdot \mathbf{p}' \eta_2}{m_2^2} + \frac{2 E^2 \mathbf{p} \cdot \mathbf{p}' \eta_2}{M_V^2} + \frac{E^2 \mathbf{p} \cdot \mathbf{p}' \eta_2}{m_2^2 M_V^2} \\
& - \frac{E^2 \mathbf{p}'^2 \eta_2}{M_V^2} - \frac{E^2 \mathbf{p}^2 \eta_2}{M_V^2} - \frac{E^2 \mathbf{p}'^2 \mathbf{p}^2 \eta_2}{m_2^2 M_V^2} + \frac{3 E^4 \eta_1^2 \eta_2}{m_1^2} + \frac{3 E^4 \mathbf{p} \cdot \mathbf{p}' \eta_1^2 \eta_2}{m_1^2 m_2^2} \\
& - \frac{2 E^4 \mathbf{p} \cdot \mathbf{p}' \eta_1^2 \eta_2}{m_1^2 M_V^2} - \frac{E^4 \mathbf{p} \cdot \mathbf{p}'^2 \eta_1^2 \eta_2}{m_1^2 m_2^2 M_V^2} + \frac{E^4 \mathbf{p}'^2 \eta_1^2 \eta_2}{m_1^2 M_V^2} + \frac{E^4 \mathbf{p}^2 \eta_1^2 \eta_2}{m_1^2 M_V^2} \\
& + \frac{E^4 \mathbf{p}'^2 \mathbf{p}^2 \eta_1^2 \eta_2}{m_1^2 m_2^2 M_V^2} - \frac{3 E^4 \mathbf{p}^2 \eta_1 \eta_2^2}{m_1^2 m_2^2} + \frac{2 E^4 \mathbf{p} \cdot \mathbf{p}' \mathbf{p}^2 \eta_1 \eta_2^2}{m_1^2 m_2^2 M_V^2} - \frac{E^4 \mathbf{p}'^2 \mathbf{p}^2 \eta_1 \eta_2^2}{m_1^2 m_2^2 M_V^2} \\
& - \frac{E^4 \mathbf{p}'^2 \eta_1 \eta_2^2}{m_1^2 m_2^2 M_V^2} + \frac{3 E^4 \eta_2^3}{m_2^2} - \frac{2 E^4 \mathbf{p} \cdot \mathbf{p}' \eta_2^3}{m_2^2 M_V^2} + \frac{E^4 \mathbf{p}'^2 \eta_2^3}{m_2^2 M_V^2} + \frac{E^4 \mathbf{p}^2 \eta_2^3}{m_2^2 M_V^2} \\
& - \frac{3 E^6 \eta_1^2 \eta_2^3}{m_1^2 m_2^2} + \frac{2 E^6 \mathbf{p} \cdot \mathbf{p}' \eta_1^2 \eta_2^3}{m_1^2 m_2^2 M_V^2} - \frac{E^6 \mathbf{p}'^2 \eta_1^2 \eta_2^3}{m_1^2 m_2^2 M_V^2} - \frac{E^6 \mathbf{p}^2 \eta_1^2 \eta_2^3}{m_1^2 m_2^2 M_V^2} \Big), \tag{A4}
\end{aligned}$$

$$K_{1S}(\mathbf{p}, \mathbf{p}') = \frac{4 c_I g_1 g'_1 2 m_1 m_2}{(m_S^2 - \mathbf{q}^2)}, \tag{A5}$$

$$K_{2S}(\mathbf{p}, \mathbf{p}') = -\frac{4 c_I g_2 g'_2 m_1 (3 m_2^2 + \mathbf{p}^2)}{3 m_2 (m_S^2 - \mathbf{q}^2)}, \tag{A6}$$

$$K_{3S}(\mathbf{p}, \mathbf{p}') = \frac{-4 c_I g_3 g'_3 \left[ m_2^2 (\mathbf{p}^2 - e e^2 \eta_1^2) + (\mathbf{p}^2 + e e^2 \eta_1 \eta_2)^2 + m_1^2 (4 m_2^2 + \mathbf{p}^2 - e e^2 \eta_2^2) \right]}{m_1 m_2 (m_S^2 - \mathbf{q}^2)} \tag{A7}$$

where  $g_1, g'_1, g_2, g'_2, g_3$  and  $g'_3$  represent the coupling constants in the vertexes and  $\mathbf{q}^2 = (\mathbf{p}^2 + \mathbf{p}'^2 - 2\mathbf{p} \cdot \mathbf{p}')$ .

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